

## New Optimal Planning of Restoration and Technical Diagnostics of Sea Berths on the Basis of Q Analysis and Fuzzy Set Theory

**David Gurgenidze\*, Merab Akhobadze\*, Malkhaz Tsikarishvili\*, Teimuraz Bulia\***

\* Faculty of Civil Engineering, Georgian Technical University, Tbilisi, Georgia

(Presented by Academy Member Tamaz Shilakadze)

**New methods of optimal planning of restoration work and technical diagnostics of sea berths showing the dynamics of damage by means of Q analysis of combinatorial topology and theory of fuzzy sets are reported in the paper. The existing methods of technical diagnostics of sea berth and structures based on the analysis of data of measuring the parameters of structures often lead to wrong results. Often the data obtained are not amenable to statistical methods of processing. The non-measurable and non-formalizable parameters of the processes effecting technical parameters of the design are not taken into account. The developed method makes it possible to overcome the problems. Based on the obtained algorithms and fuzzy technology the software package was created.** © 2023 Bull. Georg. Natl. Acad. Sci.

sea berth, fuzzy sets, Q analysis

The task of technical diagnostics of berths, building structures is to identify damage and defects in building structures, forecasting and establishing the causes of damage. Finally, making a decision on the possibility of further operation of the facility, optimal planning and implementation of restoration work [1-3].

Existing methods of technical diagnostics of berths, building structures are based on the analysis of data on measuring the parameters of structures.

Accordingly, to assess the technical condition of buildings and make decisions about its further operation, methods of mathematical statistics are used. It should be noted that the use of statistical methods in the case of fuzzy data and in tasks that cannot be fully formalized is incorrect. The fact is that the methods of mathematical statistics can be used, when the statistical data are described by a model of a random process [4].

Today, for mathematical modeling and control with fuzzy data and non-formalizable problems, the theory of fuzzy sets and fuzzy technologies is used [5].

The theory of fuzzy sets is not a fuzzy theory, but a logical interpretation of fuzziness and non-formalizable problems. When using it, the decision is made by a combination of expert knowledge and current measurements carried out at the object of study.

We can consider any building-construction, a berth as a set of a finite number of „structural elements“, „technological processes“, etc. By „structural element“, „technological process“, we mean any structural unit, means of their connection, methods, etc. Therefore, the mathematical model of a building-construction can be considered as a set of mathematical relations  $\Lambda(\lambda, \mu, \dots)$  that exist among the „structural elements“,  $(A, B, \dots)$  „technological processes“  $(S, P, \dots)$  etc. among other sets. Obviously, each „structural element“ is uniquely determined by the elements of the set  $\sigma \{k_0, k_1, \dots, k_n\}$  from which it is made. Formalization of the problem in this way allows us to consider the mathematical model of the structure as a simplicial complex, the study and analysis of which is carried out by the method of combinatorial topological analysis [8,10]. The mathematical model of a structure, presented as a simplicial complex, makes it possible to detect all kinds of connections between the structural elements of a structure and observe what quantitative and qualitative changes will be caused in „space“ time, when each „structural element“ is affected [6, 7].

The simplicial complex [8, 9] is the pair  $\bar{K} = (K, S)$ , in which  $K = \{k_0, k_1, \dots, k_n\}$  is some finite set (whose elements are called vertices of the complex  $\bar{K}$ ).  $S$  is the set of its non-empty subsets, which are called simplices of the given complex, such that:

1. For any element  $k_i \in K$ , the one-element set  $\{k_i\}$  is the simplex of this complex:  $\{k_i\} \in S$ .
2. If  $\sigma \in S$  and  $\tau \subseteq \sigma$ , then  $\tau \in S$ .

A pair of simplices  $\sigma$  and  $\tau$ ,  $q$  – connected, if  $\bar{K}$  contains a finite sequence of simplices  $\sigma, p_1, p_2, \dots, p_n, \tau$ , in which any two successive terms have a common facet of dimension at least  $q$ , i.e. They have at least  $q+1$  common vertex. Such a sequence is called a  $q$  path in  $\bar{K}$ . If two faces are  $q$  – connected, then they are also  $p$ -connected for all  $p < q$ . A complete description of the  $q$  – connection components of the complex  $\bar{K}$ , for all  $q = 0, 1, 2, \dots, \dim \bar{K}$ , is called the  $Q$  analysis of this complex.

The vector  $Q = (Q_0, Q_1, \dots, Q_{\dim \bar{K}})$ , is called  $Q$  – vector of the complex  $\bar{K}$ . It encodes information about how many  $q$  – related "parts" the complex  $\bar{K}$  consists of. Lower dimensional simplices ( $< q$ ) create "gaps" that prevent the "spread or movement of information" within the  $\bar{K}$  complex. Therefore, the  $Q$  – vector in a certain sense reflects the global geometry of the complex, its structural stability.

One of the main characteristics of the connectedness of the complex is the eccentricity, which shows how important this simplex is for the stability of the complex. Simplex eccentricity  $\sigma$  is calculated by the formula:

$$ecc(\sigma) = \frac{\hat{q} - q^*}{q^* + 1},$$

where  $\hat{q}$  is the dimension of the simplex  $\sigma (\hat{q} = \dim \sigma)$ , and

$$q^* = \max \left\{ 0 \leq i \leq \dim \sigma \mid \exists \sigma' \in \bar{K} \text{ such that } \sigma \neq \sigma' \text{ and } \sigma^{Y_i} = (\sigma')^{Y_i} \right\}.$$

Here  $\sigma^{Y_i}$  and  $(\sigma')^{Y_i}$  respectively denote the equivalence class –  $Y_i$  for  $\sigma$  and  $\sigma'$ .

A simplicial complex by a system is understood as a quadruple  $(X; Y; \rho; \pi)$ , where  $X$  and  $Y$  are sets,  $\rho \subset X \times Y$  and  $\pi$  – is a mapping  $\pi: K_\rho \rightarrow R$ , i.e., a correspondence in which each simplex corresponds

to a real number.  $\pi$  is called a model  $K_\rho$  on a simplicial complex. For example,  $\pi$  can be given by the mapping  $f : \rho \rightarrow R$ :

$$\pi[x_0; x_1; \dots; x_k] = \sum_{\forall_j (x_j; y) \in \rho} \sum_{j=0}^k f(x_j; y).$$

$f$  and  $\pi$  have the following physical content:  $f(x; y)$  it is the cost (time, money, etc.) in the system incurred to implement links  $(x; y)$  in the complex.  $\pi(\sigma)$  is the cost incurred to make all connections to  $\sigma$ . The increment  $\pi$  of the model makes it possible to optimize rehabilitation work according to various criteria [7, 8].

When representing the mathematical model of the object under study in the form of simplicial complex, to show the connection between two sets, the corresponding incidence matrix is used - a matrix, each element of which is either 1 (with a given connection between the corresponding elements), or 0 (when there is no given connection between the corresponding elements). It should be noted here that in many cases, if the relationship between individual elements of two sets is insignificant (small) compared to other relationships, or the relationship is not formalized, then for the application of  $Q$  analysis, such a relationship is considered non-existent [6, 9]. Obviously, with further use of the model obtained with such an assumption, a lot of information is lost, which can lead to incorrect results.

The method developed by us [8] allows not only to overcome this problem, but also to make the assessment of the stability of a structure (structure) more accurate than when applying the classical  $Q$  analysis. In this case, the matrix elements are not the numbers 0 and 1, a membership functions corresponding fuzzy sets.

Suppose  $A = \{a_1; a_2; \dots; a_s\}$  and  $B = \{b_1; b_2; \dots; b_s\}$  are some sets, and we characterize the relationship between these sets of a linguistic variable [10]. Denote by  $X; X^1; X^2; \dots; X^P$  fuzzy subsets corresponding to linguistic variables, a by  $\mu_p : A \times B \rightarrow [0; 1]$ , (for each  $p \in [1; P]$ ) membership function  $X^p$ . Suppose that for each pair  $(a_s; b_r)$  there is at least one  $p \in [1; P]$  such that  $\mu_p(a_s; b_r) = 1$ ; Let  $X_0^p$  denote the support of the membership function  $\mu_p$  of the corresponding fuzzy sets. The carrier of the membership function  $\mu_p$  is the set of all pairs  $(a; b) \in A \times B$  satisfying the condition:  $\mu_p(a; b) > 0$ . Since our goal is to study the relationship between different sets of the system under consideration, we consider the case when the condition is fulfilled:  $\bigcup_{p=1}^P X_0^p = A \times B$  and the sorting of sets is possible  $X^p$  levels, in other words, if  $p_1 > p_2$ , then  $X^{p_1}$  denotes a higher level of connection than  $X^{p_2}$  (for example, if we characterize the relationship between two elements of the set  $A$  And  $B$  linguistic variable "technical condition", fuzzy subsets (terms) can be:  $X^1$  - „good“;  $X^2$  - „temporarily acceptable“;  $X^3$  - „not working“).

The "fuzzy matrix" corresponding to each relation  $\Lambda$ , will be constructed as follows: for each  $s \in [1; S]$ ,  $r \in [1; R]$ , the element at the intersection of the corresponding row and column of the matrix  $\Lambda$  (denoted  $\overline{(a_s; b_r)}$  symbol) is a vector:

$$\overline{(a_s; b_r)} = (\mu_1(a_s; b_r); \mu_2(a_s; b_r); \dots; \mu_p(a_s; b_r)).$$

The coordinate of the  $Q_p$  structure vector, shows the restrictions on the exchange of information at the  $p$  level, between the elements of the set  $A^p$ . The larger  $Q_p$ , the greater the resistance at the level  $p$  for the implementation of the connection between the elements of the set  $A^p$ . The eccentricity  $p_0$  element level  $a_s$  of the set  $A^p$  is defined as follows:

$$Ecc_{p_0}(a_s) = \frac{\widehat{q}_s(p_0) - q_s(p_0)}{q_s(p_0)}.$$

The eccentricity of the element  $a_s$  at the level  $p_0$  shows the degree of „separation“, („originality“) of this element. „Feature“, the degree of „separation“ of the element  $a_s$  will be largest when  $q_s(p_0) = 0$ . When the element  $a_s$  cannot connect (exchange information) with another element even at the  $p_0$  level.

On the basis of the above methodology and algorithms, a software package for technical diagnostics of berths and structures was created, based on the statistical data of the port of Poti. Using the methods of combinatorial topology, fuzzy set theory and Fuzzy technologies, allowed, along with the statistical data obtained by measuring the structural parameters of structures, to use uncontrolled, non-formalizable factors affecting the technical condition of the berth (erosion of a monolithic concrete base; degree of corrosion of sheet pile structural parameters of structures and defects in their adhesion; damage to reinforced concrete structures of the superstructure from the effects of natural factors, etc.). To assess the technical condition of the berth, the Takagi-Sugeno method [10] was used, based on the methodology of neural networks and fuzzy logic.

### სამშენებლო მუქანიკა

**ნავმისადგომების აღდგენისა და ტექნიკური  
დიაგნოსტიკის ახალი ოპტიმალური დაგეგმარება  
 $Q$  ანალიზისა და არამკაფიო სიმრავლეთა თეორიების  
საფუძველზე**

დ. გურგენიძე\*, მ. ახობაძე\*, მ. წიქარიშვილი\*, თ. ბულია\*

საქართველოს ტექნიკური უნივერსიტეტი, სამშენებლო ფაკულტეტი, თბილისი, საქართველო

(წარმოდგენილია აკადემიის წევრის თ. შილაკაძის მიერ)

ნაშრომში წარმოდგენილია ნავმისადგომების, შენობა-ნაგებობების ტექნიკური დიაგნოსტიკის, დაზიანებათა დინამიკის პროგნოზირების და სარეაბილიტაციო სამუშაოების ოპტიმალური დაგეგმვის ახალი მეთოდი, კომბინატორული ტოპოლოგიის  $Q$  ანალიზისა და არამკაფიო სიმრავლეთა თეორიების საფუძველზე მიღებული შედეგების და Fuzzy ტექნილოგიების ბაზაზე შექმნილია გამოყენებითი პროგრამული პაკეტი.

## REFERENCES

1. Gurgenidze D., Tsikarishvili M., Bulia T. (2022) Peculiarities of technical diagnostics of berthing facilities in the port of Poti . International conference on the world practice of interdisciplinary scientific research, dedicated to the 100th anniversary of GTU, 278-290.
2. Sakvarelidze A. (2020) Ports and port facilities. Part 1, 293 p., publishing house „Technical University“. Tbilisi.
3. Bulia T., Tsikarishvili M., Akhobadze M. (2022) Forecasting the dynamics of damage to building structures, assessment of restoration work and optimal planning, q based on the method of analysis (on the example of the Poti berth). *Scientific and Technical Journal "Construction"*, 3(63): 17-27.
4. Tutubalin I.R. (1972) Theory of growing up. Moscow University Press.
5. Zade L.A. (1965) Fuzzy sets. Information and control, p. 338-353.
6. Atkin R.H. (1974) Mathematical structure in human affairs. Heinemann Educational Books LTD. London.
7. Akhobadze M., Mesablishvili B., Kurtskhilia E. (2018) Complex, macrosystems structural analysis and management.
8. Akhobadze M., Kurtskhilia E. (2022) Mathematical model of urban planning for sustainable development and urban reconstruction. *Bull. Georg. Natl. Acad. Sci.*, 16(1): 7-10.
9. Akhobadze M. (2021) Issues of mathematical modeling of macrosystems (based on the entropy maximization principle). s. 206. Tbilisi.
10. Takagi Y., Sugeno M. (1985) Fuzzy system identification and its application to modeling and control. *IEEE Trans. System Man and Cybern.*, vol. SMS-15-P. 116-132.

*Received December, 2022*